**Introduction**

The \( H_\infty \) optimal model reduction problem is that of given a complex transfer function \( G_{\text{des}} \) of order \( n \) and finding a stable reduced order transfer function \( G_r \) of order \( r \) with \( r < n \) such that the approximation error \( \| G_{\text{des}} - G_r \|_{\infty} \) is as small as possible.

\[
\begin{align*}
\| G_{\text{des}} - G_r \|_{\infty} & \leq \gamma.
\end{align*}
\]

\( H_\infty \) optimal model reduction

- In this work, a new approach for computing projection matrices is introduced.
- The projection matrices are chosen such that given one projection matrix, the other projection matrix is optimized and vice versa.
- An iterative algorithm is proposed which, given one projection matrix, optimizes the other projection matrix and then the latter projection matrix to optimize the first, and so on.

**Condition for** \( \| G_{\text{des}} - G_r \|_{\infty} \leq \gamma \).

**Fixing the signal projection matrix \( V \) and optimizing the residual projection matrix \( U \)**

Given \( G_{\text{des}} \) and \( G_r \) in (2) and (4) respectively. Let \( \gamma \geq 0 \) and let \( V \in \mathbb{R}^{n \times r} \) be a full rank matrix with \( r < n \). Then for the model reduction error dynamics the following statements are equivalent:

1. There exists \( \hat{U} \in \mathbb{R}^{n \times r} \) and \( K \in \mathbb{R}^{n \times n} \) and let \( Q = \hat{U}^T E V \), that satisfies the following LMIs.

\[
Q_1 := \begin{bmatrix}
E^T K E & 0 \\
0 & Q
\end{bmatrix} > 0,
\]

\[
Q_2 := \begin{bmatrix}
\tilde{E}^T K \tilde{E} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & C \gamma & \gamma \tilde{C} V \\
0 & 0 & \gamma \tilde{C} V & 0
\end{bmatrix} < 0.
\]

2. There exists \( U \in \mathbb{R}^{n \times r} \) such that \( G_r \) defined in (4) is stable, of order \( r \) and with \( \| G_{\text{des}} - G_r \|_{\infty} \leq \gamma \).

**Invariance of Robust stability margin**

From the small gain theorem, it is clear that robust stability margin can be kept invariant by minimizing the \( H_\infty \) norm of the reduced order model. Using the similar approach of fixing one projection matrix and optimizing for the other projection matrix with an iterative scheme, an algorithm for the invariance of Robust stability margin for descriptor systems is introduced.